# Retardation Plate Theory



## Overview

A retardation plate is an optically transparent material which resolves a beam of polarized light into two orthogonal components; retards the phase of one component relative to the other; then recombines the components into a single beam with new polarization characteristics.

## Birefringence

To better understand phase retardation, birefringence must first be discussed. Most optical materials are isotropic, i.e. having the same optical properties (and therefore one index of refraction) regardless of the direction of propagation through the material. An anisotropic material possesses different optical, electric, piezoelectric and elastic properties dependent on the orientation of the material. Crystal such as quartz, calcite and sapphire are common anisotropic materials. Polarized light propagating through such crystals will experience a different index of refraction for different directions of propagation and polarization orientations. This phenomenon is known as birefringence.

Within the material there exists a direction or axis with a unique optical property such that light propagating along it encounters only one index of refraction regardless of its polarization direction, and as such is called the optic axis.

Polarization components perpendicular to the optic axis encounter a refractive index known as the ordinary index ($n_o$), while parallel components encounter a refractive index known as the extraordinary index ($n_e$). Quantitatively, the birefringence value of a material is defined as($n_e$- $n_o$). If $n_e > n_o$ the crystal is called positive uniaxial and negative for the reverse. Often the axis which propagates the highest index value is called the fast axis.

## Retardation

We now consider the effect on polarized light due to birefringence. Polarization, being a vector quantity, can be resolved into two orthogonal components such that one experiences $n_e$ and the other $n_o$. Since the velocity of light within the crystal is inversely proportional to the index, one polarization will be traveling faster than the other.

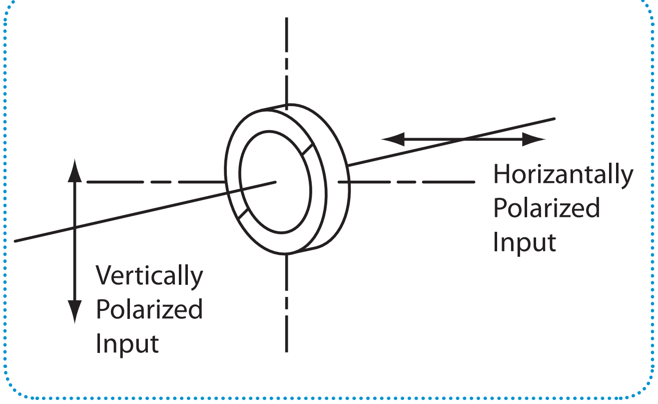
$$n_e = c / v_{\parallel} \; ; \; n_o = c / v_{\perp}$$

Where v is the velocity of light within the crystal, c is the velocity of light in a vacuum and n is the index of refraction.

As linear polarization enters a crystal, both polarization components are oscillating in phase with one another. As they propagate they begin to fall out of phase due to the velocity difference. Upon exiting, the resultant polarization is some form of elliptical polarization due to the phase difference. Special polarization cases can result if the overall phase difference is in multiples of $\pi$ (linear polarization) or $\pi/2$ (circular).

## Example

As an example consider linearly polarized light incident on a birefringent crystal with the polarization at a 45 degree angle to the fast axis (Fig. 1). In this case the magnitude of the $n_e$ and $n_o$ components are equal. The change in phase due to




the birefringence will be dependent on the plate thickness (d) of the material, as well as, the wavelength ($\lambda$) of the beam of incident light and the birefringence value. The resultant phase difference can easily be shown to be:

$$\theta = 2\pi d (n_e - n_o)/\lambda$$

where the plate thickness and wavelength are both expressed in millimeters. If the thickness of the crystal is such that a phase change of $\pi/2$ is introduced, then the resulting exiting beam will be circularly polarized. This can be shown by carrying out the vector summation of both polarization components through the use of Jones calculus. Since $\pi/2$ is equivalent to a quarter of a wave, this retarder is referred to as a quarter waveplate. Note that a change in retardance of one wave ($2\pi$) is equivalent to no change in retardance and entrance beam. Moreover if the retardance is an odd multiple of $\pi/2$, i.e. [(M+1/2)$\pi$], quarter wave retardance is achieved once again.

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The most common waveplate retardance values are $(\pi/2)$ and $(\pi)$. The quarter waveplate as previously explained will change linear polarization to circular and visa-versa. It can be shown that propagation through a half waveplates will keep linear polarization linear, however it will be rotated through an angle of $2\theta$; where $\theta$ is the angle between the incident polarization direction and the crystal's fast axis. For this reason half waveplates are often referred to as polarization rotators.

## Multiple Order Waveplates

Using the previous equation, and given a required retardance value, the necessary thickness of the waveplate can be calculated. For standard waveplate materials, such as quartz and magnesium fluoride, the calculated thickness for a retardation value of a fraction of a wave would be on the order of 0.1 mm and too thin to manufacture. For this reason a higher multiple of the required retardation is use to place the thickness of the waveplate in a physically manufacturable range. These waveplates are called multiple-order waveplates and typically have values in the neighborhood of 10 to 12 whole waves plus the fractional retardation required.

## Temperature, Wavelength and Angle of Incidence Dependence

An important consideration in using multiple order waveplate is the dependence of the retardation value on temperature, wavelength and angle of incidence. The thicker the waveplate, the higher the retardation value, and the more the retardation value will change with temperature, wavelength and angle of incidence. The change in retardation due to temperature is expressed by the following equation:

$$\Delta N_t \text{(waves)} = -0.1014N \times 10^{-3} / {}^{o}\text{C}$$

where N is the retardation value in waves.

For wavelength dependence:

$$\Delta N\ (\lambda > \lambda^{I}) = \frac{N\ \Delta n^{I}\lambda}{\Delta n \lambda^{I}} - N$$

where $\Delta n$ is the birefringence $(n_e - n_o)$.

Finally, the dependence on angle of incidence is:

$$\Delta N_t \text{(waves)} = N\phi^2 \text{ (radians)} / (2n_e n_o)$$

Obviously, from the preceding equations, the smaller the value of N the less effect the temperature, wavelength, and the angle of incidence will have on the retardation value. This is key disadvantage of a multiple order waveplate.

## Zero Order Waveplates

Based on the above equations it would be highly advantageous to construct a waveplate of zero order i.e. M=0 and N equals exactly one quarter or one half wave thickness. This can be achieved if the plate is constructed of two plate halves which have their fast axes crossed. The thicknesses are chosen so that the difference in retardation between the two plates is equivalent to the desired retardance. Any variation in temperature, wavelength, or angle of incidence dependence is greatly reduced. This type of waveplate is called a zero-order waveplate. Plate halves can be cemented, optically contacted or air-spaced.

## Achromatic Waveplates

The wavelength dependence of the birefringence dictates that the spectral range for the standard waveplate is approximately 10 nm. To accommodate tunable sources or sources with larger spectral widths, a waveplate that is relatively independent of wavelength is required This is accomplished with achromatic waveplates.

An achromatic waveplate is a zero-order waveplate utilizing two different birefringent materials. As a result of the dissimilar wavelength dependence value shifts in one plate due to the wavelength change will be compensated by the other plate. It can be shown that an achromatic waveplate can be constructed by using the following equations:

$$d_a = \frac{N(\lambda_1 \Delta n_{2B} - \lambda_2 \Delta n_{1b})}{\Delta n_{1a}\Delta_{2b} - \Delta n_{1b}\Delta n_{2a}} \qquad d_b = \frac{N(\lambda_2 \Delta n_{1a} - \lambda_2 \Delta n_{2a})}{\Delta n_{1a}\Delta_{2b} - \Delta n_{1b}\Delta n_{2a}}$$

where $d_a$ and $d_b$ are the material thicknesses, N is the desired retardation value, $\lambda_1$ and $\lambda_2$ are the wavelength extremes and $\Delta n_{ij}$ are the birefringence values at the different wavelengths. The resulting retardance can be fairly constant over a range of several hundred nanometers. The most common achromatic waveplates material combination is quartz and magnesium fluoride.

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